

The Neutrino Option

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The minimal seesaw scenario can radiatively generate the Higgs potential to induce electroweak symmetry breaking while supplying an origin of the Higgs vacuum expectation value from an underlying Majorana scale. If the Higgs potential and (derived) electroweak scale have this origin, the heavy $SU(3) \times SU(2) \times U(1)_Y$ singlet states are expected to reside at $m_N \sim 10 - 500$ PeV for couplings $|\omega| \sim 10^{-4.5} - 10^{-6}$ between the Majorana sector and the Standard Model. In this framework, the challenge of the electroweak scale hierarchy problem is replaced with a need to generate or accommodate PeV Majorana mass scales in ultraviolet models; the usual hierarchy problem is absent as the electroweak scale is not a fundamental scale.

I. Introduction. The Standard Model (SM) provides a successful description of many particle physics measurements but does not explain the experimental evidence for dark matter and neutrino masses. These facts argue for extensions of the SM that experience some form of decoupling through small couplings, and/or a ratio of scales in an Effective Field Theory (EFT) setting [1] so that the corrections to the SM in the SM effective field theory (SMEFT) framework are small perturbations.

Minimal (viable) extensions of the SM are usually beset with an inability to address the Electroweak (EW) scale hierarchy problem. A SMEFT statement of this problem is that threshold corrections to $H^\dagger H$ can be generated by integrating out sectors extending the SM proportional to large scales $\Lambda \gg m_h$. Without parameter tuning the Higgs mass is expected to be proximate to the cut off scale of the theory for this reason. Symmetries, such as supersymmetry, can be invoked to suppress these threshold corrections. This is frequently done while assuming the EW scale is a fundamental parameter and the SM Higgs potential has a classical form that leads to $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$ symmetry breaking. As a result, one generally expects new particles related to stabilizing symmetries, such as superpartners, to be around the TeV scale and found at LHC to avoid fine tuning. Unfortunately, this has not (as yet) occurred.

In this paper we do not adopt the assumptions that the EW scale and Higgs potential are fixed classically without a dynamical origin. We develop an alternative approach using the minimal seesaw scenario [2–5] as an extension of the SM. The idea is that the Higgs mass and potential are generated by threshold corrections from the Majorana [6] sector and the anomalous breaking of scale invariance in the Coleman-Weinberg (CW) potential [7]. The result is an origin of the observed neutrino masses within a SM extension that avoids parameter tuning. An origin for the EW scale is introduced that obviates the usual concerns about an EW scale hierarchy problem. The purpose of this paper is to demonstrate this possibility – “the neutrino option”.

The scenario presented is falsifiable and depends in a sensitive manner on the measured value of the top quark and Higgs masses, and the detailed neutrino mass spectrum. Higher order Renormalization Group Equations (RGEs) and threshold matching calculations are also critical in this scenario. This approach strongly motivates more theoretical and experimental progress in all of these areas in order to falsify or confirm this possible origin of the Higgs potential and EW scale, as we show.

II. Seesaw model and threshold corrections.

We use the formalism for the seesaw extension of the SM in Refs. [8, 9]. The extension of the SM Lagrangian with $p, q = \{1, 2, 3\}$ singlet fields N_p is given by

$$2\mathcal{L}_{N_p} = \overline{N_p}(i\not{\partial} - m_p)N_p - \overline{\ell}_L^\beta \tilde{H} \omega_\beta^{p,\dagger} N_p, \quad (1)$$

$$- \overline{\ell}_L^{c\beta} \tilde{H}^* \omega_\beta^{p,T} N_p - \overline{N_p} \omega_\beta^{p,*} \tilde{H}^T \ell_L^{c\beta} - \overline{N_p} \omega_\beta^p \tilde{H}^\dagger \ell_L^\beta.$$

The $\omega_\beta^p = \{x_\beta, y_\beta, z_\beta\}$ are each complex vectors in flavour space. These vectors have absorbed the Majorana phases θ_p . The mass eigenstate Majorana fields are defined such that they satisfy the Majorana condition [6]: $N_p^c = N_p$. These fields are related to chiral right handed fields N_R that are singlets under $SU(3) \times SU(2)_L \times U(1)_Y$ as [9]

$$N_p = e^{i\theta_p/2} N_{R,p} + e^{-i\theta_p/2} (N_{R,p})^c. \quad (2)$$

The superscript c stands for charge conjugation; defined on a four component Dirac spinor as $\psi^c = -i\gamma_2 \gamma_0 \overline{\psi}^T$. Integrating out the seesaw model at tree level, the results can be mapped to the SMEFT up to dimension seven [8]. The threshold corrections of interest lead to dimension two and four (SM) terms in the SMEFT Lagrangian. They come about due to integrating out the heavy N_p states at one loop and running down to the scales that are used to experimentally probe the SM states and interactions in the EW vacuum. The diagrams of Fig.1 give a threshold matching to the Higgs potential terms

$$V(H^\dagger H) = -\frac{m_p^2 |\omega_p|^2}{16\pi^2} F_1 H^\dagger H - \lambda_{pq} F_2 (H^\dagger H)^2, \quad (3)$$

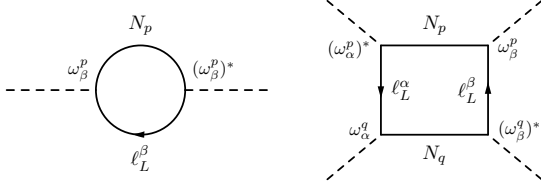


FIG. 1: One loop corrections matching onto and generating the Higgs potential at the scale(s) m_p in the seesaw model.

where

$$F_1 = 1 + \log \frac{\mu^2}{m_p^2}, \quad (4)$$

$$F_2 = 1 - \frac{m_p m_q \log \frac{m_p^2}{m_q^2} + m_q^2 \log \frac{\mu^2}{m_q^2} - m_p^2 \log \frac{\mu^2}{m_p^2}}{(m_p^2 - m_q^2)}, \quad (5)$$

and $\lambda_{pq} = 5(\omega_q \cdot \omega^{p,*})(\omega_p \cdot \omega^{q,*})/(64\pi^2)$. Here the repeated indices are summed over and the results can be compared to past results in Ref. [10, 11]. We have used dimensional regularization in $d = 4 - 2\epsilon$ dimensions and $\overline{\text{MS}}$ here and below. The counterterms of the SM and the full theory including the N_p states cancel the ϵ divergences in each case. The mismatch of the SMEFT Lagrangian and the full theory in the limit $p^2/m_p^2 \rightarrow 0$ (with kinematic invariants denoted p^2) defines the threshold matching. The renormalization scale dependent logs in the result can be neglected when also neglecting the (net) two loop effects due to running between the threshold matchings.

Considering the parameterization of the Higgs potential (V_c) as

$$V_c(H^\dagger H) = -\frac{m^2}{2}(H^\dagger H) + \lambda(H^\dagger H)^2, \quad (6)$$

neglecting the effect of running down from the scale(s) $\mu = m_p$, and assuming the mass differences are parametrically smaller than intrinsic mass scales in the Majorana sector ($(m_q^2 - m_p^2)/m_{p,q}^2 < 1$) we find

$$\Delta m^2 = m_p^2 \frac{|\omega_p|^2}{8\pi^2}, \quad \Delta \lambda = -5 \frac{(\omega_q \cdot \omega^{p,*})(\omega_p \cdot \omega^{q,*})}{64\pi^2}. \quad (7)$$

Note that the λ threshold corrections can be subdominant to other quantum corrections in the full CW potential in the parameter space of interest where $|\omega_p| \ll 1$ and $m_p \gg 246 \text{ GeV}$.

III. Induced CW potential. The threshold corrections to $H^\dagger H$ can be naturally dominant in defining the Higgs potential below the scales $\mu \simeq m_p$. The reason is that the SM is classically scale invariant in the limit that the vacuum expectation value (vev) of the Higgs $v \rightarrow 0$ [7, 14–16] ($m \rightarrow 0$ in Eqn. 6). This point of enhanced symmetry is anomalous, even before its soft

breaking by the threshold matching. However, the additional SM breakings of scale invariance through quantum corrections are associated with dimensionful parameters that are smaller than m_p^2 in a consistent version of this scenario at the threshold matching scale.

A breaking of the scaleless limit of the SM is due to QCD, which generates the scale Λ_{QCD} by dimensional transmutation [7] at low scales as $(\Lambda_{QCD}/\mu)^{b_0} = \text{Exp}[-8\pi/\hbar g_3^2(\mu)]$ where $b_0 = 11 - (2/3)n_f$ [12, 13]. The quark masses that result lead to V_c contributions such as

$$\Delta m^2 = \frac{N_c y_t^2 \Lambda_{QCD}^2}{32\pi^2} \left(1 + 3 \log \left[\frac{\mu^2}{\Lambda_{QCD}^2} \right] \right) + \dots \quad (8)$$

which subsequently induces a vev for the Higgs, leading to gauge boson masses $\propto \Lambda_{QCD}$. As we are assuming $m_p^2 |\omega_p|^2 \gg \Lambda_{QCD}^2$ (for each p) these contributions are naturally subdominant for $H^\dagger H$, and anyway, at the threshold matching scale we consider, the QCD coupling has run to scales such that $g_3(m_p) < 1$. Renormalization of the CW potential also introduces an anomalous breaking of scale invariance. Consider defining $V_{CW}(\langle H^\dagger H \rangle)$ as the one loop CW potential expanding around the scaleless limit of the SM while neglecting the threshold corrections. The standard result [7, 14, 15] can be minimized via $\partial V_{CW}/\partial \langle H^\dagger H \rangle = 0$. The vev scale obtained is exponentially separated from the renormalization scale. This scale is associated with the asymptotic nature of the perturbative expansions used in constructing the CW potential, that also predict S -matrix elements that are used to fix SM (or SMEFT) Lagrangian parameters. This scale can be either suppressed or enhanced depending on the net sign of the quantum correction in the CW potential, and a suppression is consistent with an EFT analysis.

In summary, the soft breaking of the scaleless limit of the SM¹ is such that the threshold corrections to $H^\dagger H$ due to integrating out the N_p states can be a dominant contribution to V_{CW} fixing a high scale boundary condition for the Higgs potential. This occurs for interesting parameter space when tuning of the threshold corrections against bare parameters is avoided expanding around the classically scaleless limit of the SM Lagrangian.

IV. Running down to the scale $\mu^2 = \hat{m}_t^2$. Assuming that the Higgs potential is (dominantly) given by Eqn. 7 when integrating out the Majorana sector that includes the N_p states, a non trivial consistency condition of this scenario is the successful generation of the

¹ Previous studies of the CW potential in this scaleless limit (not advocating the Neutrino Option for the Higgs potential) include Refs. [16–23]. Note that we have introduced a hard breaking of scale invariance due to m_p in Eqn. 1. If such masses were spontaneously generated the breaking of scale invariance would be completely spontaneous and introduce a dilaton [24] in the spectrum. This scenario is beyond the scope of this work. We thank D. McGady for conversations on this point.

SM potential at lower energy scales and field values. The measured masses of the SM states and their couplings ensures this can occur in a very nontrivial fashion. Note also that despite the fact that the seesaw boundary condition fixes $\lambda < 0$, this coupling can still run to positive values at lower scales as it is not multiplicatively renormalized.

To demonstrate that a successful lower scale phenomenology can result from these seesaw boundary conditions, we take $V_c(H^\dagger H)$ to be fixed to $m^2(m_p) \equiv \Delta m^2$ and $\lambda(m_p) \equiv \Delta\lambda$. The parameters m^2 and λ are then run down according to the coupled SM RGEs. The β -functions for running above the top mass are introduced as $\beta(x) = (4\pi)^2 dx/d\ln\mu^2$ and are taken (at leading order) from the summary in Ref. [34] as

$$\begin{aligned} \beta(g_Y^2) &= g_Y^4 \frac{41}{6}, \quad \beta(g_2^2) = g_2^4 \left(-\frac{19}{6}\right), \quad \beta(g_3^2) = g_3^4(-7), \\ \beta(\lambda) &= \left[\lambda \left(12\lambda + 6Y_t^2 - \frac{9}{10}(5g_2^2 + \frac{5}{3}g_Y^2) \right) \right. \\ &\quad \left. - 3Y_t^4 + \frac{9}{16}g_2^4 + \frac{3}{16}g_Y^4 + \frac{3}{8}g_Y^2g_2^2 \right], \end{aligned} \quad (9)$$

$$\beta(m^2) = m^2 \left[6\lambda + 3Y_t^2 - \frac{9}{20}(5g_2^2 + \frac{5}{3}g_Y^2) \right], \quad (10)$$

$$\beta(Y_t^2) = Y_t^2 \left[\frac{9}{2}Y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_Y^2 \right]. \quad (11)$$

Here g_2, g_3 are the coupling constants of the $SU(2)_L$ and $SU(3)_c$ gauge groups, while g_Y the coupling of the $U(1)_Y$ group. $Y_i = \sqrt{2}m_i/v$ is the Yukawa coupling of a fermion to the Higgs with $v^2 = 1/\sqrt{2}\hat{G}_F$. Contributions proportional to Y_b and Y_τ have been neglected. The differential system is solved by fixing the boundary conditions to be

$$\lambda(m_p) = -n^2 \frac{5}{64\pi^2} |\omega|^4, \quad m^2(m_p) = \frac{n|\omega|^2}{8\pi^2} m_p^2, \quad (12)$$

$$\hat{Y}_t(m_t) = Y_t^{(0)} + Y_t^{(1)}(m_t) = 0.9460, \quad (13)$$

$$\hat{g}_Y(m_t) = g_Y^{(0)} + g_Y^{(1)}(m_t) = 0.3668, \quad (14)$$

$$\hat{g}_2(m_t) = g_2^{(0)} + g_2^{(1)}(m_t) = 0.6390, \quad (15)$$

$$\hat{g}_3(m_t) = 1.1671, \quad (16)$$

for different choices of m_p and $|\omega|$ which approximate to one common universal scale and coupling in what follows. The number of heavy neutrino species has been denoted with n and fixed to $n = 3$. The $x^{(0)}$ and $x^{(1)}(\mu)$ stand for the tree and one-loop level contribution in the SM and we have denoted as hatted quantities observables inferred from measured input parameters. The analytic expressions for the tree-level definitions used are standard for the $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ input parameter set. The central values of the numerical inputs used are

$$\{\hat{m}_Z, \hat{m}_W, \hat{m}_t, \hat{m}_h\} = \{91.1875, 80.387, 173.2, 125.09\},$$

in GeV units, $\hat{G}_F = 1.166378710^{-5} \text{ GeV}^{-2}$ and $\hat{\alpha}_s = 0.1185$. The expressions for the $x^{(1)}(\mu)$ used are sum-

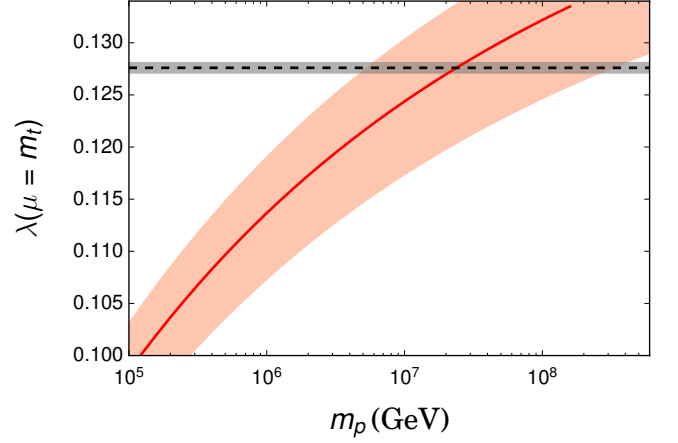


FIG. 2: Value of the parameter λ extrapolated at the scale $\mu = \hat{m}_t$ as a function of the heavy neutrino mass scale m_p . The red line assumes $\hat{m}_t = 173.2$ GeV. The surrounding band corresponds to varying \hat{m}_t between 171 and 175 GeV (a 2σ error variation [25]). The dashed line and surrounding band marks the value of $\lambda(m_t)$ computed in the SM and its percentage error [26].

marized in Ref. [34]. The value indicated for the QCD coupling g_3 includes RG running at 4-loops in QCD and 2-loops in the EW interactions employing the expression as a function of $\alpha_3(m_Z)$ and m_t as in Ref. [34]. The quantity $\lambda(\mu = m_t)$ does not show significant dependence on the parameter $|\omega|$: for any value $|\omega| < 0.1$ we have $|\Delta\lambda| \lesssim 10^{-6}$, which is numerically insignificant in the running. Conversely, this quantity is quite sensitive to the scale m_p and the RGE order used. There is significant numerical sensitivity to the input parameters. In particular, the precise experimental determination of $\{\hat{m}_h, \hat{m}_t\}$ is critical for the consistency of the scenario. We show the dependence on these critical measured inputs on the inferred scale m_p in Fig. 2. This plot shows the value for $\lambda(\hat{m}_t)$ consistent with experimental measurements is obtained for $m_p \simeq 10^{1.3}$ PeV, assuming $\hat{m}_t = 173.2$ GeV. The quantity $m^2(\mu = \hat{m}_t)$ is sensitive to both m_p and $|\omega|$. Fig. 3 shows its dependence on $|\omega|$ for the fixed value $m_p = 10^{1.3}$ PeV and the corresponding viable band associated to the uncertainty on the top mass determination.²

This scenario can lead to a SM-like Higgs potential emerging from the combined effect of the threshold corrections and the SM RGEs in a nontrivial fashion as shown in Fig. 4.

V. Cosmological and low energy constraints. The sum of the observed neutrino masses is $\sum_i m_\nu^i \simeq$

² This interesting region of parameter space in the seesaw model has been previously discussed in Ref. [27].

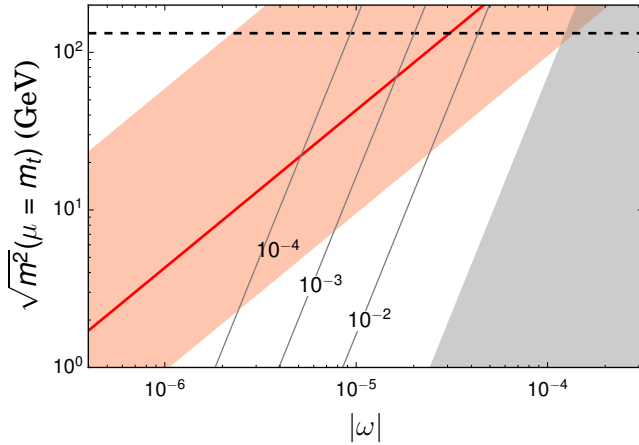


FIG. 3: Value of $\sqrt{m^2}$ extrapolated at $\mu = \hat{m}_t$ as a function of $|\omega|$. The dashed black horizontal line indicates the value consistent with the measured Higgs mass, while the red solid line is obtained for $m_p = 10^{1.3}$ PeV. The red shaded region corresponds to the uncertainty on the top quark mass, consistent with Fig. 2 and the grey region is disfavoured due to Λ CDM cosmology limits on the sum of neutrino masses as given in Eqn. 17. Finally the neutrino mass scales predicted (in eV) are the three solid lines.

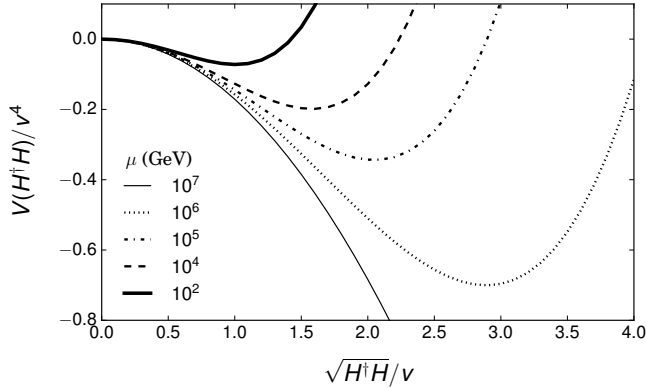


FIG. 4: The emergence of the Higgs potential due to running the seesaw boundary conditions down to $\mu \sim \hat{m}_t$.

$3|\omega|^2/2\sqrt{2}\hat{G}_F m_p$ in the tree level approximation used here, while neglecting running effects. Assuming a Λ CDM cosmology, combined CMB, supernovae and Baryon Acoustic Oscillation data limits this sum [28]. This translates into a constraint of

$$\frac{3\sqrt{3}}{8\pi}\frac{|\omega|^2}{\hat{G}_F\sqrt{\Delta m^2}} \lesssim 0.23 \text{ eV}, \quad 95\% \text{ C.L.}, \quad (17)$$

which is shown as a grey exclusion region on Fig. 3.

The overall neutrino mass scale predicted in the scenario is very sensitive to the uncertainty on \hat{m}_t , the chosen order of RGEs, and threshold loop corrections included in the numerical simulation. The measured mass

differences for the neutrino masses are given as [25]

$$\Delta m_{21}^2/10^{-5} \text{ eV}^2 = 6.93 - 7.97, \quad (18)$$

$$\Delta m^2/10^{-3} \text{ eV}^2 = 2.37 - 2.63 (2.33 - 2.60) \quad (19)$$

using PDG notation. Here the quoted range is 3σ and the brackets indicate an inverted m_ν hierarchy. In our simple approximation of a common universal scale and coupling for the N_p states integrated out, we do not predict mass differences of the m_ν . The mass eigenstate differences are related to the weak eigenstate masses predicted in the seesaw model through rotation matrices contributing to the PMNS mixing matrix (see Ref.[8] for details). In Fig. 3 we show the absolute neutrino mass scales (grey lines) predicted at leading order as $|m_\nu| = 3|\omega|^2/2\sqrt{2}\hat{G}_F m_p$. One expects $|m_\nu|^2 \gtrsim \Delta m_{21}^2, \Delta m^2$ to avoid fine tuning and a requirement of further model building in the Majorana sector.

In addition, a negative sign for λ and m^2 indicates a theory with a Hamiltonian unbounded from below. However, the corresponding decay time for the EW vacuum is exponentially small [30–33]. We have checked that the EW vacuum decays in this scenario are well approximated by the (negligible) result in the SM in Ref. [34]. The ratio of the scales at which $\beta(\lambda)$ vanishes in the SM, compared to the SM extension considered in this paper, (which fixes the size of the action of the bounce) is ~ 1.00011 . The extrapolation the theory far above the scale m_p is associated with a large theory uncertainty as the N_p could be embedded in an extended Majorana sector, with other states that can also modify the running of the couplings above the scale $\mu \simeq m_p$.

VI. Numerical stability of the results. The results shown in Section IV are produced with one loop matching conditions and one loop RGEs. Increasing the RGE and threshold matching order used shows significant numerical sensitivity. This is essentially because the coupling λ is running to small and negative values asymptotically which introduces a sensitivity to the scale m_p where the seesaw boundary conditions are matched to. This feeds into the required $|\omega|$ to produce the Higgs potential and EW scale, and subsequently the predicted neutrino mass scale. For this reason, the minimal scenario is falsifiable. On the other hand, the uncertainty in \hat{m}_t introduces a significant numerical uncertainty in the results. To illustrate this we show in Fig. 5 the best fit points for the cases where the boundary conditions of the scenario are evolved with one loop SM RGEs, two loop SM RGEs, and one loop RGEs for Δm and λ and two loop RGEs for the remaining SM parameters.³ This

³ Formally the running should be described using the SMEFT RGEs which include the effect of higher dimensional operators feeding into the running of the SM couplings [29]. We have

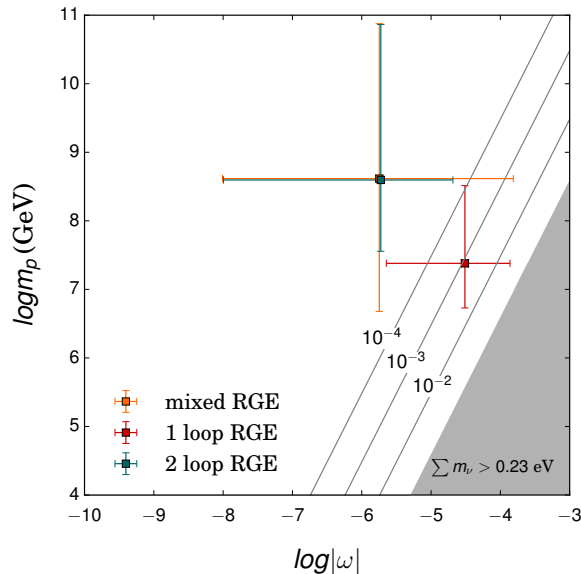


FIG. 5: Numerical sensitivity of the results, with all cases showing one loop matching to Δm , λ (due to the model assumption) and including one loop corrections for the remaining SM parameters with one, two loop, or mixed RGEs for all parameters. The mixed case shows one loop running for Δm , λ and two loop running for all remaining SM parameters. The best fit points are indicated with a box in each case with error bars showing the experimental uncertainty in the top quark mass, which has been chosen to be its 2σ uncertainty [25].

last case is shown as these parameters do not have a tree level matching coefficient in this scenario.

VII. Tree level decays and IceCube. The tree level decays of the N_p states are well known, see Refs. [35, 36]. Remarkably, the mass range selected for when the Higgs potential is radiatively generated in the minimal seesaw scenario is consistent with the measured energies of an excess of neutrinos reported by IceCube [37–39]. The $d\Gamma/dE_\nu$ spectrum that results from these decays is a sharp mono-chromatic peak at the scale $m_p/2$.

The possibility that the seesaw scenario states N_p can be a viable dark matter candidate to induce the IceCube events has been examined in the literature. We agree with the results of Refs. [40–42] that the required coupling for the event rate scales as $\Gamma_{events} \sim (|\omega|/10^{-29})^2 (m_p/1.2\text{PeV})/\text{year}$ which is inconsistent with the $|\omega|$ preferred to generate the Higgs potential in the minimal seesaw model with a PeV Majorana sector. Extended model building can possibly accommodate these observations.

checked that this effect is numerically sub-dominant in this model and neglected it.

VIII. Conclusions. Due to a non-trivial interplay of the couplings of the SM and the mass scales of the SM states expanded around the classically scaleless limit, the minimal seesaw scenario can form a UV boundary condition that induces the Higgs potential at lower energies. This can occur as a simple mechanism to generate neutrino masses is introduced extending the SM, as experimentally required.

In this scenario the EW scale is not fundamental but is due to the quantum threshold corrections matching the heavy singlet states onto the SMEFT, which is assumed to be expanded in its near scaleless limit. Instead of an expectation of new states at the TeV scale, the multi-PeV scale is the locus of a requirement of a mass generating mechanism for Majorana states, and possibly accompanying stabilizing symmetries such as supersymmetry. This change in perspective on the hierarchy problem is possibly valuable. The key point underlying this approach is to abandon attempts to stabilise the Higgs mass against threshold corrections at the TeV scale, due to the lack of experimental indications of new states associated with stabilizing symmetries. Instead we advocate embracing these corrections as the origin of the EW scale and Higgs potential. This approach can also be developed in other models beyond the minimal scenario considered here.

Future experimental results supporting this scenario are a continued lack of discovery of states motivated by a traditional interpretation of the hierarchy problem at the LHC, and the eventual discovery of the Majorana nature of neutrinos in $0\nu\beta\beta$ decay. The scenario can also be tested through consistency tests of the neutrino mass spectrum and the PMNS matrix due to a minimal seesaw scenario and more precise measurements of \hat{m}_t, \hat{m}_h at LHC. In this manner the Neutrino Option in generating the Higgs potential can be experimentally falsified. Further phenomenological investigations, and the continued advance of higher order SM threshold and RGE calculations are also strongly motivated.

Acknowledgements MT and IB acknowledge generous support from the Villum Fonden and partial support by the Danish National Research Foundation (DNRF91). We thank Cliff Burgess, Jacob Bourjaily, Poul Damgaard, Gitte Elgaard-Clausen, Belén Gavela, Yun Jiang, Jason Koskinen, David McGady, Philipp Mertsch, André Mendes and Jure Zupan for helpful discussions, and André David for technical assistance. MT thanks the organizers of EW-Moriond 2017 for an inspiring environment as this paper was completed.

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